DRAFT

# FDM Solution

Objective was to have an accurate, fine grid to compare the NN solution to. To quantify, did convergence study by looking at pointwise discrepancy between grids increasing in resolution. These were compared by restricting the higher resolution grid. Wanted to also try interpolating the lower resolution grids into higher resolution but would’ve been more time-consuming and had a fine enough grid already. As data was only needed once there was no need to not use the highest resolution grid obtained.

Equation for L2

Two graphs, noting 6400 means L2 diff from 3200 to 6400 grid.

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| Figure – L2 Error vs Grid Size, noting grid size indicates length of square array (NxN). Further reduction in points didn’t compute solution. |

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| Computational time taken vs grid size. Dotted line is quadratic best fit approximation (function shown). |

When viscosity was changed, FDM solution had to be recalculated. Instead of repeating the above, grid size of 6400x6400 was chosen and run. For lower viscosity (, run time increased a substantial amount (from 37 minutes for the previous 6400 grid to 54 minutes).

Increasing to a much higher viscosity led to a case with no shock forming. This case was not looked at further.

# Burgers (Overview)

3 hyperparameters investigated: Training point number, shape of neural network (mainly node number) and convergence criteria (factr). Then viscosity was changed, changing the equation to be solved. A new answer was computed using FDM as described above, and the training point investigation was repeated.

# NN Training points

Again how 2 or more runs for each point might be useful. The anomaly run 3 times where less training points much faster than more. Also, might the standard deviation be used for making error bars? Lastly, the trained model was used to project onto a 6400x6400 grid. Would training point < 6400 have an impact as a result?

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| Figure – Training point number investigation, for viscosity of . Investigation at 2500 training point was re-run to quickly test for variability and it was seen that the NN’s capabilities could vary substantially, with one re-run achieving even better results than versions with more training points (1E-3 mean error vs 2E-3). | |

# NN Shape

Default shape was 3 dense layers of 32, 16 and 32 nodes respectively. This investigation was carried out by halving the number of nodes and looking at the effect on the quality of the solution. Raissi’s paper mentions they utilised 6 layers of 20 nodes, so this was also looked at. In order to quantify this, the number of parameters (total connections between nodes) was plotted against the mean error. For reference, the 6x20 case had 3021 parameters compared to the default case’s 1201 parameters.

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| Figure – Note log scale on mean error | |

For figure on right, time taken seems to approach a finite number with increasing parameter number, but might be that the higher layer ‘20s’ case is more efficient computationally, giving it the wrong shape. For a relatively small increase to time taken, the 6 layer case has lower error than the other cases (note log scale)

# NN factr (convergence criteria)

Looked at high and low training point, to better understand when it plays a factor/ whether the impact varies depending on other factors. Looks like a linear relationship in time, harder to tell with so few points/repetitions for mean error. However, it’s clear that with more points gradient of time taken higher, as a result sacrificing factr yields much greater time saves. At the same time, more training points means error still lower for modest factr.

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| Figure – Time taken vs factr, and mean error vs factr. Note lower number = more strict convergence criteria (in theory more accuracy). 1E1 = highest accuracy, 1E7 = modest, 1E12 = low. | |

# Increased Viscosity

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| Time taken vs training points used | Mean error measured vs training points used |

Comparing 15k\_0025 viscosity to a case with similar error (2.5k\_001viscosity) visually:

